**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:** Since work begins 10 minutes after the car is dropped, the time left to complete work is 50 mins. The probability that the Service Manager cannot meet his commitment =P(X>50) =1-Pr(x<=50) (X is the time taken to complete work). Convert 50 to z-score.

Standard normal variable Z= (X- *μ/σ =(x-45)/8*

P(X<=50) =P (Z<= (50-45)/8) =PR(Z<=0.625) =0.73237=73.237% (the number in z-table)

The probability that the service manager will not meet his commitment is 100-73.237=26.763%= 0.2676 So, the answer is B.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:** μ=38 & *σ* = 6

1. Probability of employees >44- Prix>44)-1-Pr(x=44)

Z=(X-μ)/ σ = (x-38)/6

Pr(x+44) =Pr(2c+(44-38)/6)-Pr(Z=1) =0.84134-84.134%

Probability that employees will be greater than 44-100-84.134-15.866

Probability of employees between 38 & 44- Prix<=44)-Pr(x=38)

Here, Prtxc-44) =0.84134

Pr(x38)-Prize>-38-38)/6)-Pr(z>=0)-05

Therefore, Prix-44)-Pr(x-38)-0.84134-0.5-0.34134-34.134% So, the statement "More employees at the processing center are older than 44 than

between 38 and 44" is TRUE

1. Probability of employees less than 30 = Pr(x < 30) z = (x - 30)

Z=(x-μ)/ σ=(30-38)/6

Pr(x < 30) = Pr(z < 30 - 38) / 6 )=Pr(z<-1.333)=.09176=9.1796

So, the number of employees with a probability of 0.0917 of being under

30=400^ \* 0.0917 = 36.68 = 36

The statement of "training program for employees under the age of 30 at the center would

be expected to attract about 36 employees is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans**: X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e. Z is a linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore, in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

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4. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

**Ans**: Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of a random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting a value between a and b should be 0.99.

So, the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans**: Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company.

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans**: The 5TH Percentile of profit for the company is 17 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year

**Ans**:  The Division #2 (Profit2 ~ N(7, 42) ) has a larger probability of making a loss in a given year